

The Impact of Systematic Uncertainties

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For the DUNE Collaboration





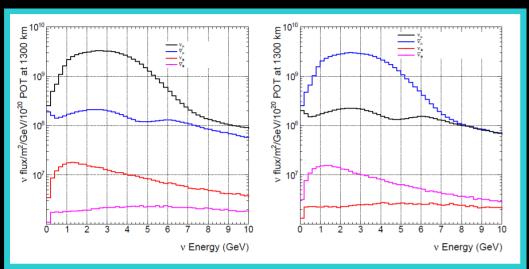
NuFact 2015

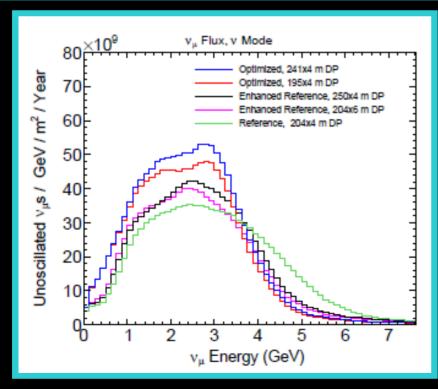
August 10 – 15, 2015 Centro Brasileiro de Pesquisas Físicas Rio De Janeiro, Brazil

Outline

- The DUNE experiment
- Expected FD spectra
- Sensitivities and systematics in the DUNE CDR
- Capabilities of a DUNE FD only fits & propagating detailed systematic uncertainties
- Program to constrain systematic uncertainties
- Propagating constraints from the DUNE ND

The DUNE Experiment





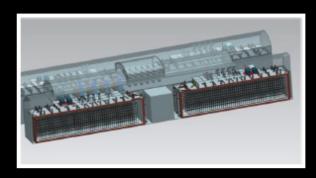
LBNF

- Built and operated by FNAL
- Beam
 - Wideband beam, peaked at 2.5 - 3.0 GeV
 - Uses 60 120 GeV protons from the Main Injector
 - PIP II upgrades enable a 1.2 MW beam
 - Upgradeable to 2.4 MW
 - Ongoing optimization of target, horns, etc to improve flux rates and shape
- Conventional facilities
 - Near Detector complex
 - Far Detector complex

The DUNE Experiment

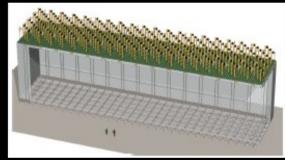
DUNE

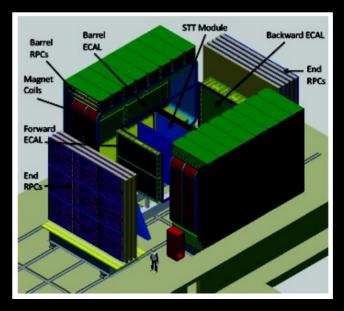
- The experimental collaboration
- Responsible for building an operating the Near and Far detectors
- Baseline: 1300km
- Exposure: 300 600 kton·MW·yr
- Far Detector
 - 40 kton LArTPC
 - Single or dual phase design
 - Staged construction
- Near Detector
 - Fine grained tracker (FGT)
 - Low density
 - Superior PID
 - High energy and angle resolutions



Dual-Phase 10 kton module

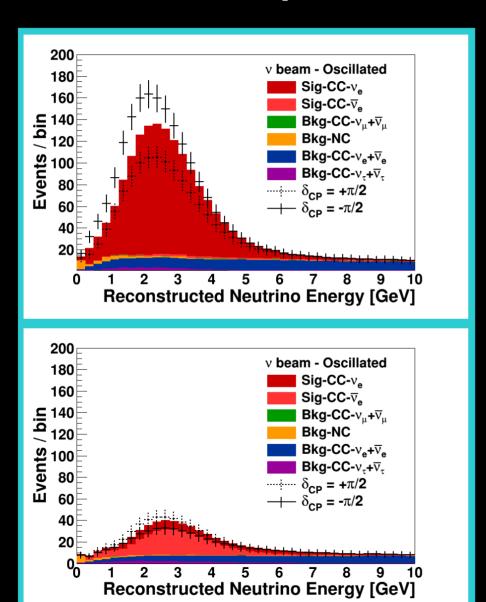
Single-Phase 10 kton module





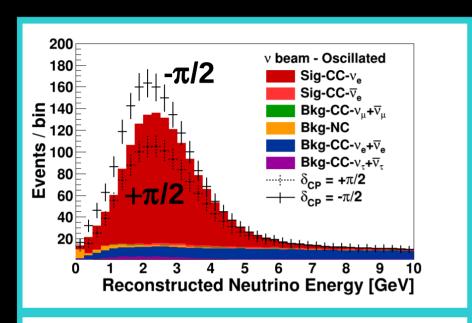
FGT Near Detector

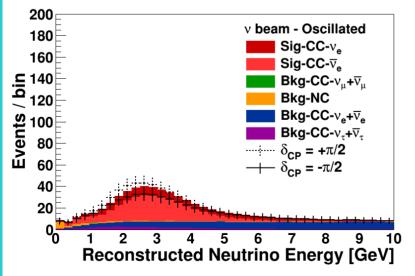
Expected FD Spectra



- Spectra produced by a Fast MC
- Fast MC inputs:
 - Full G4LBNE flux simulation
 - GENIE cross sections and FSI
 - Parameterized detector response applied to individual particles that exit the nucleus
 - Event selection based on PID of lepton candidates
- Fast MC outputs (all event-by-event):
 - Reconstructed quantities e.g. E_v, Q², W^2 , x, y, etc
 - Etrue → Ereco smearing functions
 - Efficiencies for signal and backgrounds
 - Weights for most sources of systematic uncertainty and spectral response **functions**

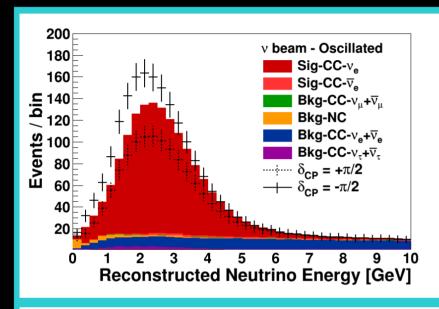
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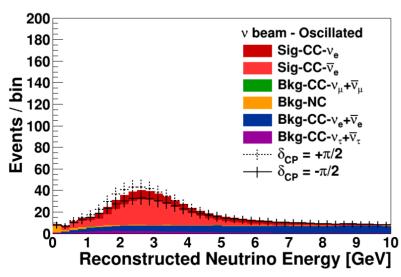


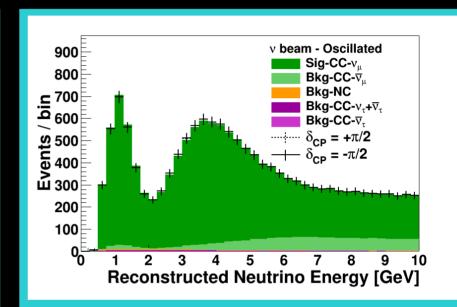


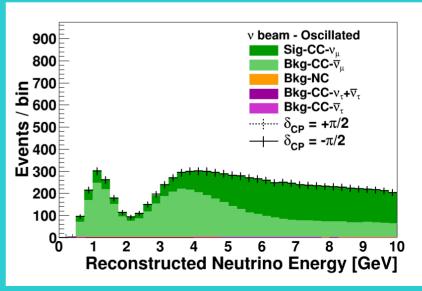
- Assumed exposure:
 - 40 kton LAr TPC FD
 - 1.2 MW beam
 - NuMI style horns
 - 120 GeV protons
 - Many possible optimizations
 - 6 yr v / 6 yr $\sqrt{}$ (56% up time)
- Oscillation Parameters
 - NuFit 2014 NH results
 - Choose $\delta_{co} = 0$
- Opposite effects on ν and ν spectra for $\delta_{cp} \rightarrow \pm \pi/2$

Expected FD Spectra

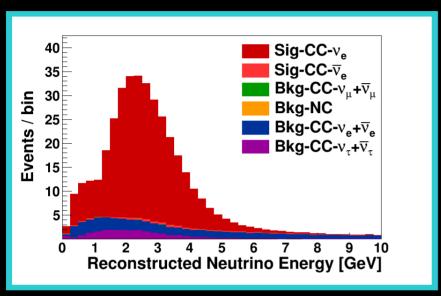




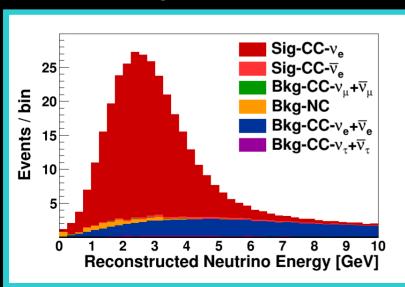




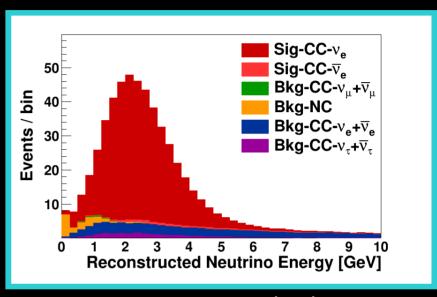
Spectra By Cross Section Model



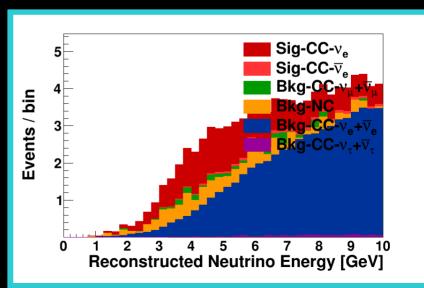
Quasi-elastic



DIS (W < 2.7 GeV)

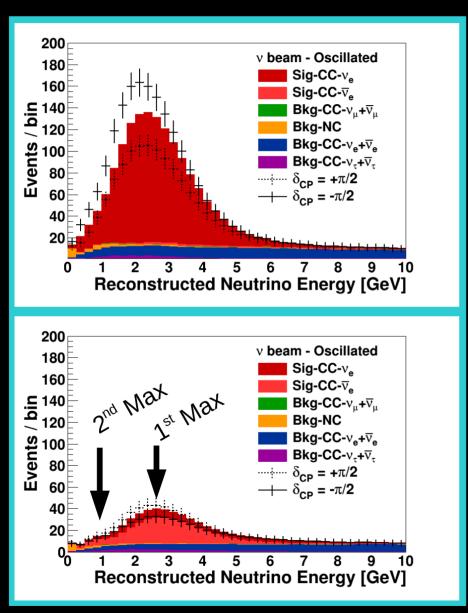


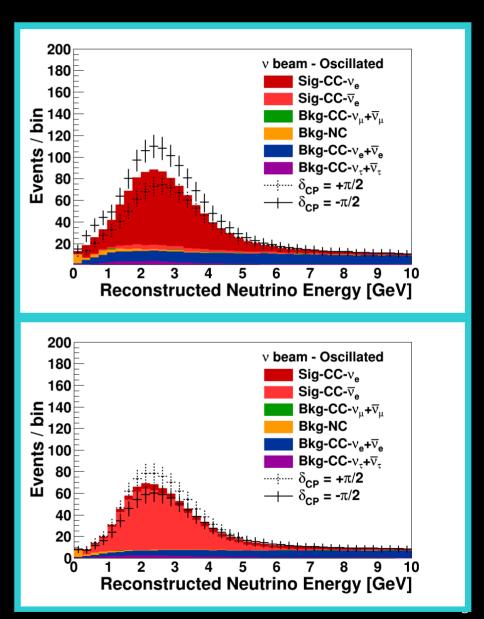
Resonance Production



DIS (W > 2.7 GeV)

Spectral Differences: v_e Appearance





Normal Hierarchy

Inverted Hierarchy

Determining CDR Sensitivities

• Define CPV sensitivity as:

$$\Delta \chi^2_{\text{CPV}} = \text{Min}(\chi^2_{\text{test}}(\delta_{\text{cp}}=0), \chi^2_{\text{test}}(\delta_{\text{cp}}=\pi)) - \chi^2_{\text{true}}$$

Define MH sensitivity as:

$$\Delta T_{NH(IH)} = \chi^2_{IH(NH)} - \chi^2_{NH(IH)}$$

- Use Asimov data sets; gives mean $\Delta \chi^2$
- Allow oscillation parameters, and systematics to vary
 - Constrain oscillation parameter values with NuFit2014 results; use $1/3^{rd}$ of the 3 σ ranges
 - Estimate non-oscillation systematics with normalization parameters
 - Consider channel-to-channel and sample-to-sample correlations

Signal uncertainties of 5% on v_{μ} disappearance and 5 \oplus 2% on v_{e} appearance assume a relative calibration in the 4-sample fits

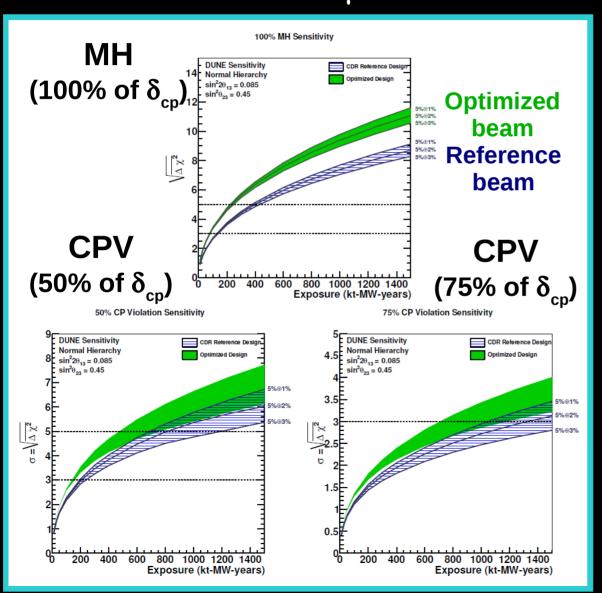
| Background | Normalization Uncertainty | Correlations | | |
|--|---------------------------|---|--|--|
| For $ u_e/ar{ u}_e$ appearance: | | | | |
| Beam $ u_e$ | 5% | Uncorrelated in $ u_e$ and $ar u_e$ samples | | |
| NC | 5% | Correlated in $ u_e$ and $ar u_e$ samples | | |
| $ u_{\mu}$ CC | 5% | Correlated to NC | | |
| $ u_{	au}$ CC | 20% | Correlated in $ u_e$ and $ar u_e$ samples | | |
| For $ u_{\mu}/ar{ u}_{\mu}$ disappearance: | | | | |
| NC | 5% | Uncorrelated to $ u_e/ar u_e$ NC background | | |
| $\nu_{	au}$ | 20% | Correlated to $ u_e/ar{ u}_e u_	au$ background | | |

Normalization uncertainties

- Estimate uncertainties after ND and external data constraints
- Understand advantages of LAr TPC, and cancellations in FD 4sample fits
- Consider experience from T2K and MINOS
 - MINOS similarities
 - Flux shape, v energies
 - Longer baseline
 - Similar cross sections
 - T2K similarities
 - Different near and far detector technologies
 - Similar analysis strategies
 - Strategies to address required increase in precision

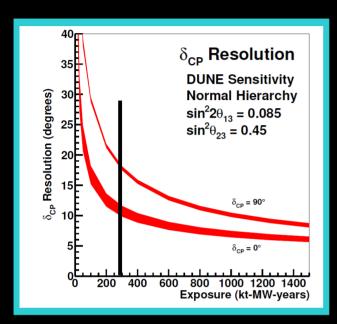
| Source of | MINOS | T2K | DUNE |
|---------------|--------|----------|------------------|
| Uncertainty | $ u_e$ | $ u_e$ | $ u_e$ |
| Beam Flux | 0.3% | 3.2% | 2% |
| after N/F | | | |
| extrapolation | | | |
| Interaction | 2.7% | 5.3% | $\sim 2\%$ |
| Model | | | |
| Energy scale | 3.5% | included | (2%) |
| (u_{μ}) | | above | |
| Energy scale | 2.7% | 2.5% | 2% |
| (ν_e) | | includes | |
| | | all FD | |
| | | effects | |
| Fiducial | 2.4% | 1% | 1% |
| volume | | | |
| Total | 5.7% | 6.8% | 3.6 % |
| Used in DUNE | | | $5\% \oplus 2\%$ |
| Sensitivity | | | |
| Calculations | | | |

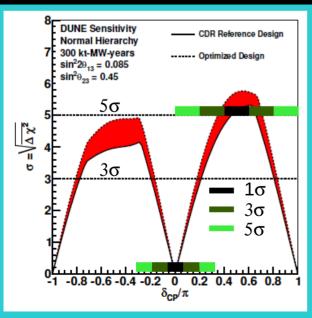
Effects of Changing the Relative v_e to v_{μ} Uncertainties

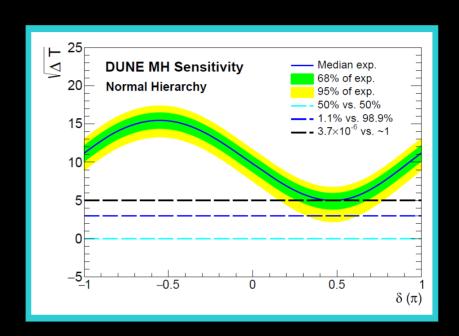


- Increased relative uncertainty barely effects MH determination
- The effect on CPV sensitivity is greater, esp at the peaks
- Beam optimization is as important as systematic uncertainty reduction

Understanding Sensitivities







- Careful attention must be paid to the statistics of MH determination (above)
- CPV sensitivity can be understood by considering the resolution on δ_{cp} (left)

Far Detector Capabilities

- The FD analysis will be preformed with 4(+) analysis sample
 - v_e appearance
 - $-\overline{v}_{e}$ appearance
 - v_u disappearance
 - $-\overline{v}_{u}$ disappearance
- Shifts in δ_{cp} will effect each of these samples differently
- Systematics will often effect all 4 samples similarly
- Combined fits to the 4 samples will implicitly constrain many sources of systematic uncertainty
- Dangerous systematics must be able to mimic the effects of shifting δ_{cp} for all 4 samples
- Need the ability to propagate detailed uncertainties to fits
- Studies are not to determine *if* a ND is needed, but to understand the design requirements to ensure it is able to compliment the capabilities of the FD

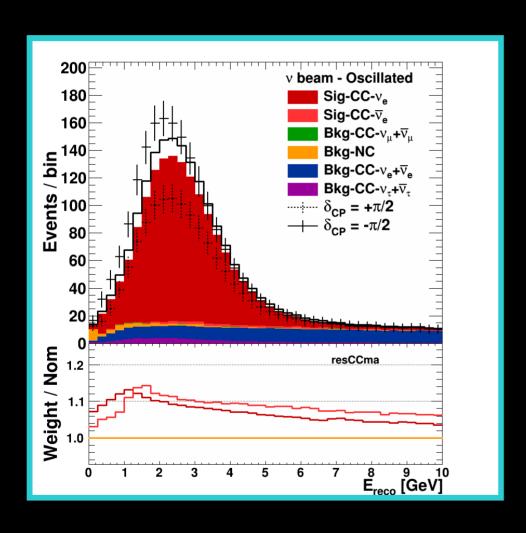
Sources of Uncertainty

- Oscillation Parameter Uncertainties (NuFit14) & Exotics
- Flux (alter G4LBNE parameters)
- Cross section models (GENIE)
- Nuclear models (Intranuke, or absorbed in cross sections)
- Detector response and reconstruction (lepton/hadron, bias/spread)
- Projecting uncertainties to the DUNE error can be difficult
 - Relatively new (far) detector technology
 - Beam and ND design have yet to converge
 - Broad R&D research program is just getting underway
 - More data will help ... unless of course there is tension between results and/or with theoretical predictions and generators

Uncertainty "Highlights"

- For systematics to be dangerous they must be able to replicate the effects of shifting δ_{co} in all 4 analysis samples
- Absolute flux normalization and shape
 - Secondary and tertiary hadron production
 - Flux shape differences at the Near and Far detectors
- Uncertainties from cross section models and nuclear initial state models need to be factorized
- A coherent picture of nuclear initial state effects is required
- Cross section flavor differences and rates for exclusive final state channels require theoretical input
- The convolution of flux, cross section, FSI and detector effects in determining energy scale will be difficult to untangle
 - Both FSI and detector effects can be different for v and \overline{v}
 - Relative \overline{v}/v uncertainties currently provide freedom to mimic δ_{cp} -like effects
- Biases in the energy scale from mis-reconstruction and/or poorly modeled/constrained missing energy (neutrons) must be eliminated

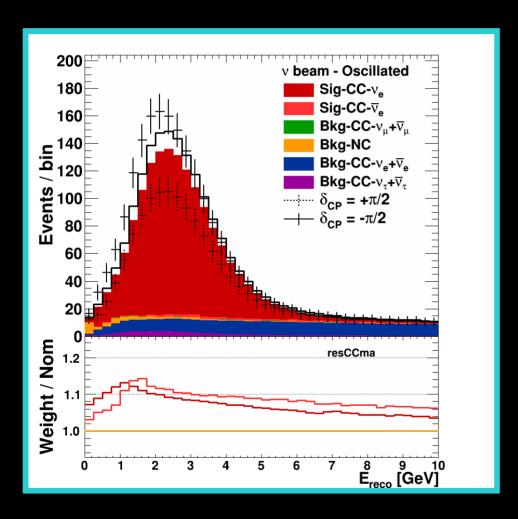
Propagating Individual Systematics

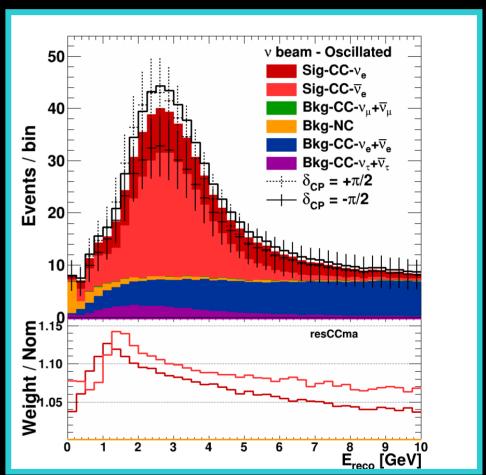


- For example:
 - Fluctuation of M_A^{res} by +1 σ
 - Induces an effect similar to changing $\delta_{\mbox{\tiny cp}}$
- However ...
 - The effect on \overline{v}_e appearance from changes in M_A^{res} is the same
 - But the effect of the same shift in δ_{cp} is opposite
 - Also the high statistics ν_{μ} disappearance sample will help constrain no effect from δ_{cp}

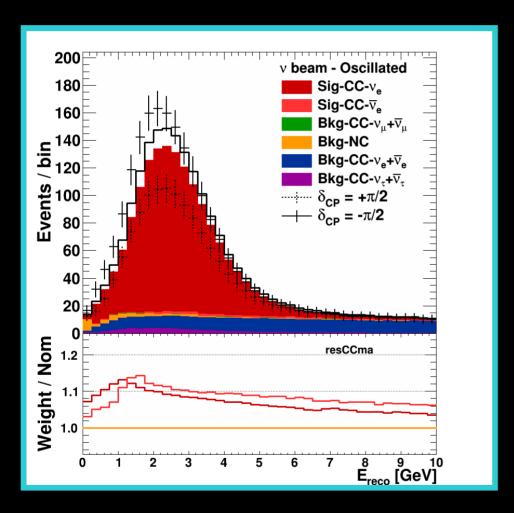
[†] Systematics are propagated to spectra via 'response functions' calculated from Fast MC event weights

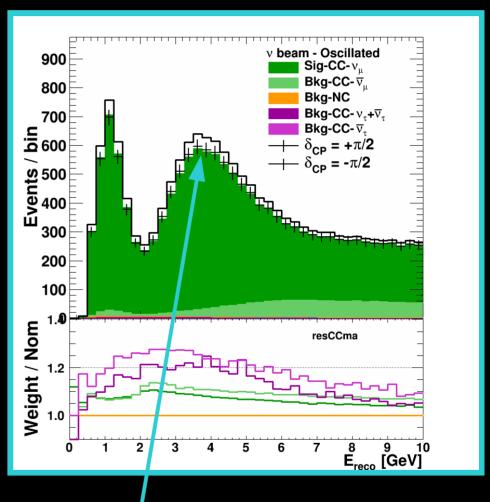
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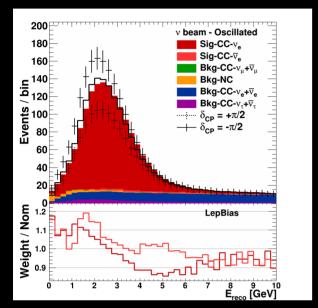


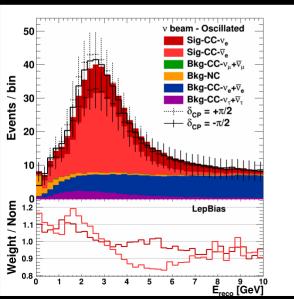
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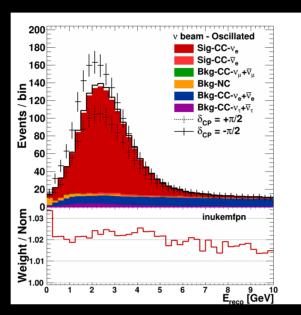


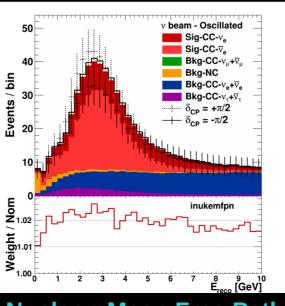
Far Detector Capabilities





Lepton E-scale Bias (2.5%)





Nucleon Mean Free Path

- We See this same behavior for many systematics
- How correlated are these effects among samples?
- Must consider:
 - Cross section ratio constraints
 - Differences in detector response
 - Statistical power of dominant constraint
 - NC/CC, v_e / v_μ , v_τ / v_μ , $\frac{1}{v}$ / v_μ

Cross Section Ratio Uncertainties

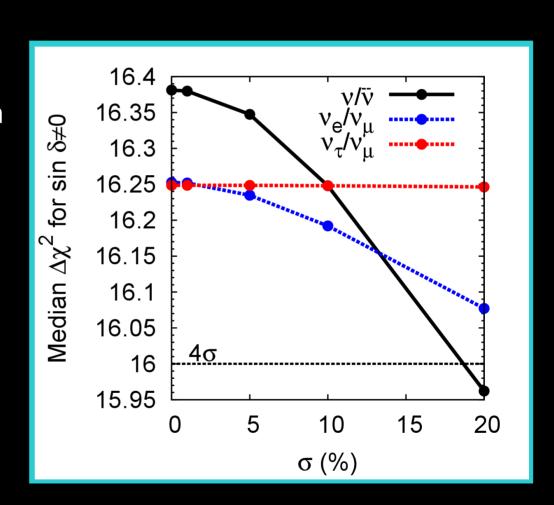
- All fits include cross section ratio uncertainties
- The uncertainty on each ratio can be set individually
- So far, no energy dependence allowed
- Default values:

$$- \sigma(\overline{v}/v) = 10\%$$

$$- \sigma(v_e/v_u) = 2.5\%$$

$$- \sigma(v_{\tau}/v_{u}) = 10\%$$

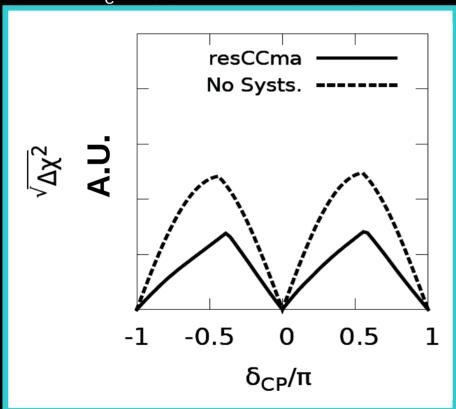
- Can study the effect of changing the values for each parameter
- Additional fit parameters to include statistical limits of constraints



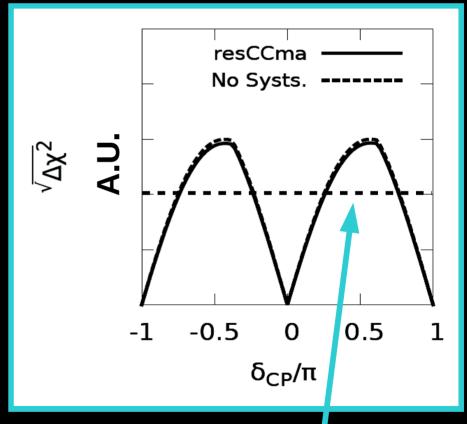
Example: CC M_Ares

Sensitivity to CPV with Variations in CC M_A res

 v_{e} - appearance only



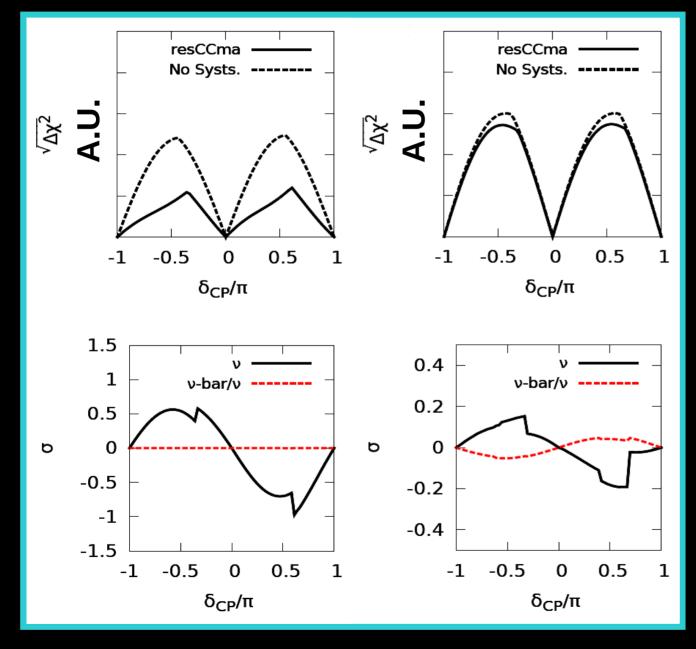
All 4 samples



- No oscillation parameter uncertainties
- FD only fits (no ND constraints)
- Allow CC M_a^{res} to vary by GENIE 1σ (±20%)

Metric: loss of CP fraction at some C.L.

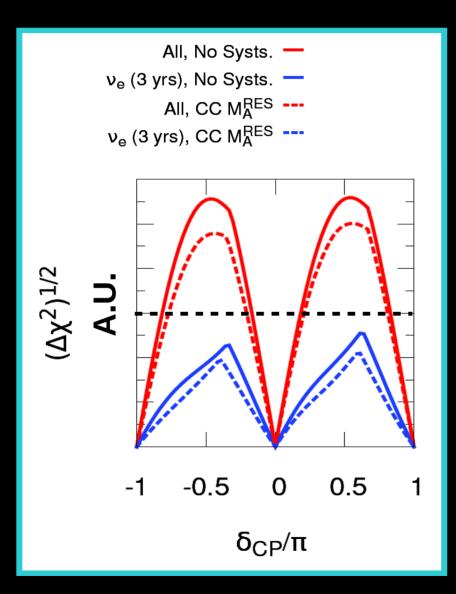
Pull Terms for CC M_A res



- For v_e only fit (left) the pull on CC M_A^{res} is up to $\sim 0.5 1.0 \sigma (10-20\%)$
- The combined fit (right) limits the variation to $\sim 0.2\sigma$ (4%)
- v/v difference allowed; error on M_A^{res} absorbs nuclear effects
- Multiple systematics may introduce additional freedom

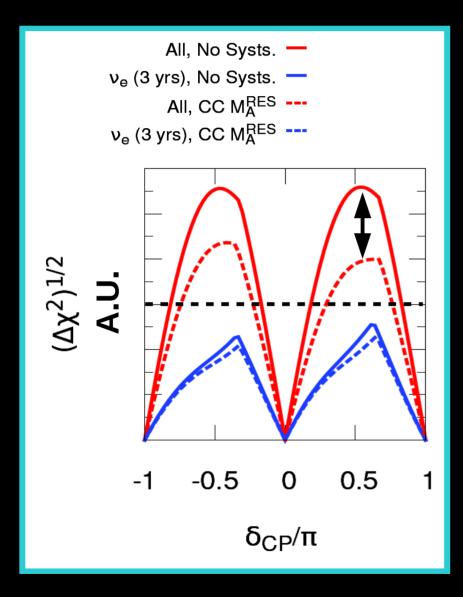
† Oscillation uncertainties included here

CPV Sensitivity with Variations in Cross Section Systematics



- Include several cross section systematics
 - $-M_A^{QE}$
 - $-M_A^{res}$
 - Resonance → DIS transition region
 - Intranuclear rescattering (FSI) parameters
- Include oscillation parameter uncertainties
- Cross section ratio uncertainties considered
- FD only fits (no ND constraints)
- Overall sensitivity degradation is still fairly small

CPV Sensitivity with Variations in Flux Systematics



- Include several flux systematics related to beam optics
- Does not include hadron production systematics
- Include oscillation parameter uncertainties
- Cross section ratio uncertainties considered
- FD only fits (no ND constraints)
- Larger sensitivity degradation
- ND MUST constrain the flux

Constraining the Flux

- Using the DUNE ND
 - 2.5% Absolute Flux (0.5 10 GeV)
 - e-v NC cross section
 - Low, well constrained bkg
 - E_{v} limited to ~13% by intrinsic $v p_{T}$
 - 3% Absolute Flux (10 50 GeV)
 - e-v CC cross section
 - 20% well constrained bkg
 - 1-2% Relative Flux (0.5 50 GeV)
 - Low-ν₀ method
 - Very low proton threshold (low-v₀)
 - Uncertainty dominated by E_μ resolution
 - Relative FHC/RHC flux
 - Coherent interactions have same cross section for v and \overline{v}

- Beamline monitoring
 - Muon monitors
 - Hadron monitors
- External Data
 - Hadron production
 - Thin target, thick, and/or replica target
 - Data from NA41, NA61, and MIPP
 - Still hard to constrain secondary and tertiary reactions
- Challenges:
 - Modeling of Far/Near spectral differences
 - Intrinsic $v p_T$ at the ND

Constraining Cross Section Models

• The DUNE ND

- High precision flux measurements remove leading error source
- High statistics inclusive samples across all v flavors + NC
- Multiple nuclear targets including
 ⁴⁰Ar
- Superior Vertex resolution:
 - Sub mm resolution, multi-track events
 - Statistical subtraction, single track events
- Reduce impact via reduced background acceptances

External data

- FNAL INP will measure low energy cross sections in LAr TPCs
- CAPTAIN Minerva will measure high energy event vertices on LAr, with downstream tracker
- Electron scattering data on Ar from JPARC will help constrain nuclear models

Challenges:

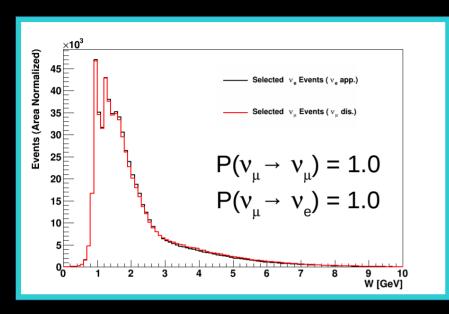
- v_e/v_μ and v_τ/v_μ cross section ratios
- Distinguishing initial and final state nuclear effects
- FSI differences for v and v

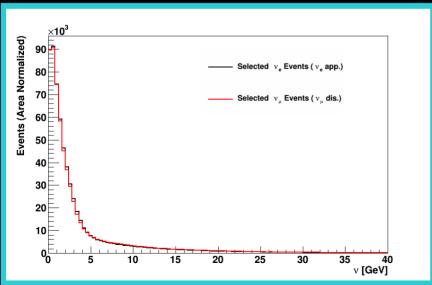
Final State Interactions (FSI)

- External measurements
 - N/π scattering off Ar
 - Already lots of data
 - Compare simulations (GENIE vs GiBUU)
- Test Beam measurements
 - p/π energy resolutions and detection thresholds
 - Detector response to n
- Neutrino beam measurements
 - Vertex activity
 - Rate and angular distribution of nucleons
 - In situ neutron counting

- Strategy to untangle FSI effects
 - FSI for ν_{μ} and ν_{e} are the same
 - Oscillation minima are the same for $v_{\rm u}$ and $\overline{v}_{\rm u}$
 - For δ_{cp} = [0, π] oscillation min/max are the same for ν_{μ} and ν_{e}
 - The appearance max shifts with δ_{cp}
 - Look for a relative shift in ν_e/ν_μ and an opposite shift for $\bar{\nu}_e/\bar{\nu}_\mu$
 - Understand absolute difference by requiring the ν_μ and $\bar{\nu}_\mu$ minima to match
 - Requires W and v for v_{e}/v_{μ} to be similar

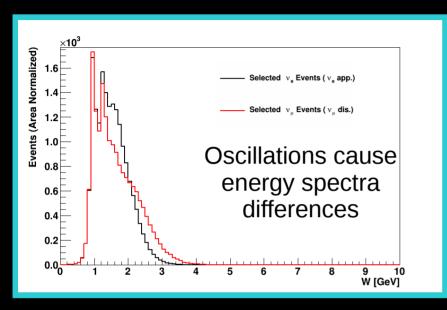
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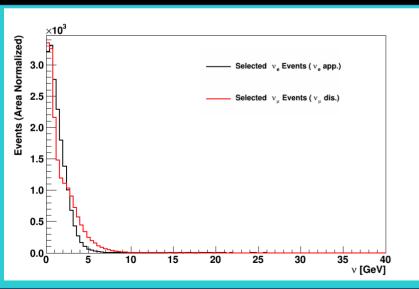




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Final State Interactions (FSI)



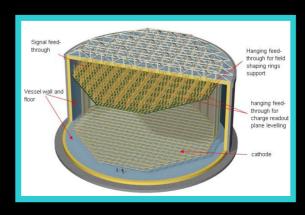


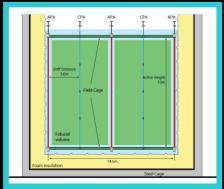
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Constraining the Energy Scale

- Test beam measurements
 - CERN Prototypes (below)
 - Both single- and dual-phase
 - Charged particle beam
 - Detector response and energy calibration reduces energy scale uncertainties
 - CAPTAIN
 - Test LAr TPC neutron response
 - What fraction of neutrons do deposit observable energy?
 - What fraction of the neutron energy is deposited?
 - What is the time structure?
 - Can we apply some neutron energy calibration?

- In-situ FD calibration
 - Atmospheric muons
 - Source of MIPs
 - Stability over time and position
 - Analysis spectra comparisons
- Split v_e appearance samples
 - QE-like(1/3) and non-QE (2/3)
 - Use QE kinematic reco.
 - Tight cuts on QE-like sample





Calculating ND Constraints

ND Fast MC

- Simulation:
 - FGT response based on NOMAD
 - dE/dx as a function of KE from G4 simulation
- Studies of flux and cross section analyses
 - Realistic selections give signal efficiencies and background rates
 - Estimates statistical strengths of these measurements
 - Demonstrates methods for constraining nuclear effects
- Next steps
 - Evaluate systematics
 - Determine potential correlations from combined fits

VALOR

- Full ND+FD fitting oscillation analysis software developed for T2K
- Applied to LBNE, LBNO, AND T2HK simulations
- Combined fit of multiple topologically defined samples
- Fit parameters related to flux, cross section, and detector response, each with a prior
- Most parameters well constrained
- Next steps
 - Apply to latest DUNE simulations
 - Include alternate ND configurations

Concluding Remarks

- There are a large number of systematic uncertainties to consider
- The FD can constrain many systematics itself with 4-sample fits
- There is a comprehensive program underway to understand and constrain many sources of systematic uncertainty – especially LAr TPC cross section and detector effects
- The DUNE ND will provide excellent flux and cross section constraints
- There is a lot of work to be done to determine the impact of each systematic and each component of the DUNE experimental setup
 - Need to estimate and propagate each uncertainty
 - Independent program of study required to ensure systematic uncertainty estimates give proper coverage
- Systematic goals have been set, and design decisions will be made to execute those goals

Backup Slides

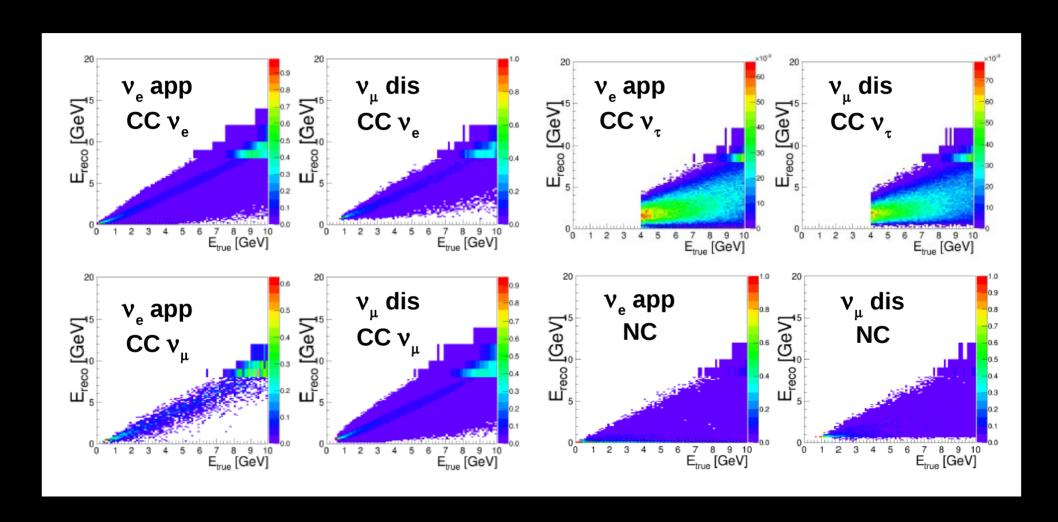
The Deep Underground Neutrino Experiment

New international science collaboration formed in late 2014 with the submission of an LOI (https://indico.fnal.gov/getFile.py/access?resId=0&materialId=4&confld=9013)

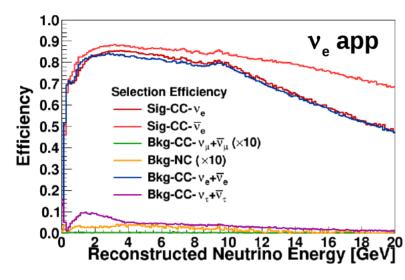


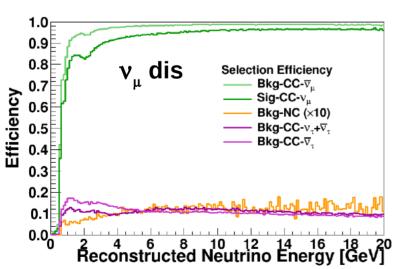
- → February 2015 collaboration meeting at FNAL
- → 776 Collaborators → 26 countries
- → 144 institutions → Members from LBNE, LBNO and more
- → Recently passed CD1-refresh review of technical design

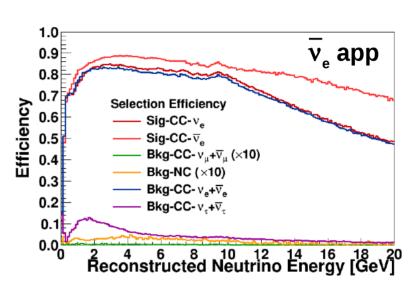
Fast MC Output: Energy Smearing

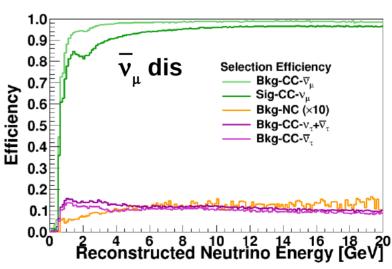


Fast MC Output: Efficiencies

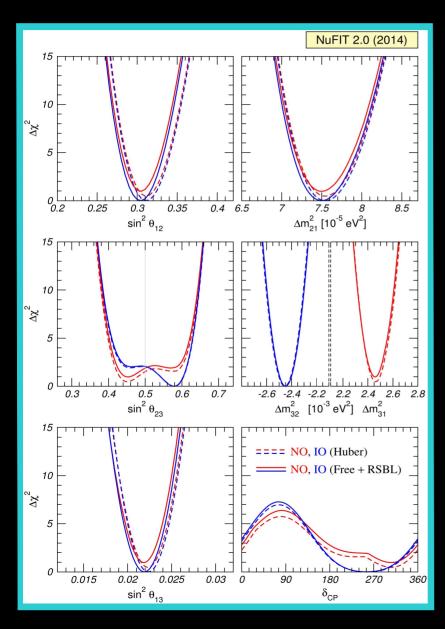








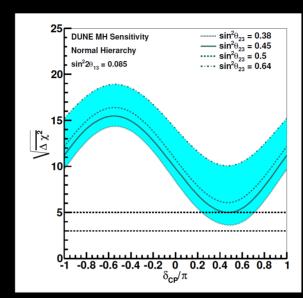
Oscillation Parameter Uncertainties and Exotic Models

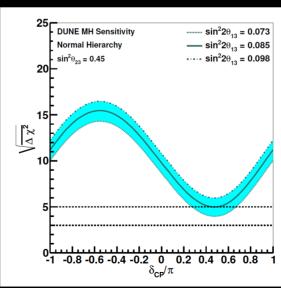


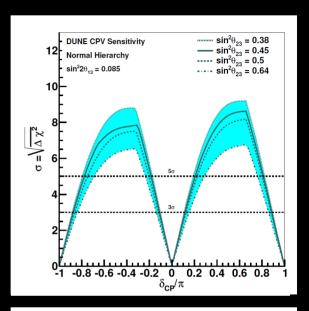
- Currently θ_{23} has the largest uncertainties
 - Has the largest effect on MH and $\delta_{\mbox{\tiny cp}}$ measurements
 - Unknown octant (is θ_{23} > or < 45°)
- Solar oscillation parameters have effects at lower energies, near 2nd oscillation maximum
- Degeneracies between $\delta_{\mbox{\tiny cp}}$ and the MH reduce sensitivity
 - $+\delta_{cp}/$ NH and $-\delta_{cp}/$ IH are produce similar spectra
 - $-\delta_{\rm cp}$ / NH, and $+\delta_{\rm cp}$ / IH are easily distinguished
- Extraction of δ_{cp} and the MH assume the canonical 3-flavor model
 - Several "model extensions" that can alter analysis spectra expectations
 - Sterile neutrinos
 - Non-standard interactions
 - Non-unitarity

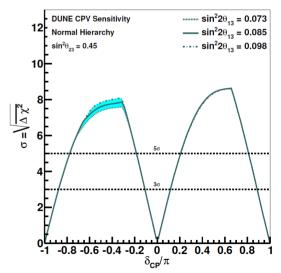
Oscillation Parameter Uncertainties

- The MH determination (left) is
 - highly dependent on the true value of $\sin^2\theta_{23}$ (top)
 - Less dependent on true $\sin^2\theta_{13}$ (bottom)
- CPV (right) sensitivity has a similar dependence
- MH determination is easier for a high $\sin^2\theta_{23}$, while CPV sensitivity is best for low values of $\sin^2\theta_{23}$



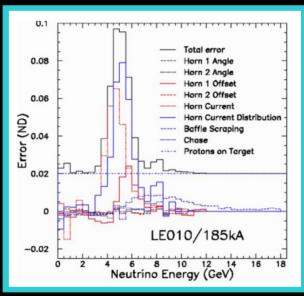


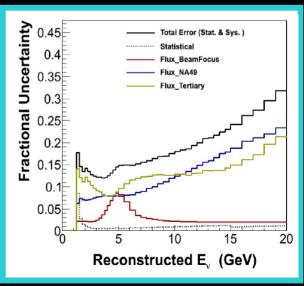




Flux Uncertainties

- Uncertainties in the beamline optics can mostly be constrained by beamline monitoring and ND data
- Hadron production modeling uncertainties are the leading source of flux uncertainties
 - Primary interactions in the target are constrained by data
 - Secondary and tertiary interactions are much more difficult to model and constrain
- Flux uncertainties are the leading source of error in many cross section measurements
- Uncertainties from all sources are routinely encoded in a covariance matrix in bins of true E_{ν}



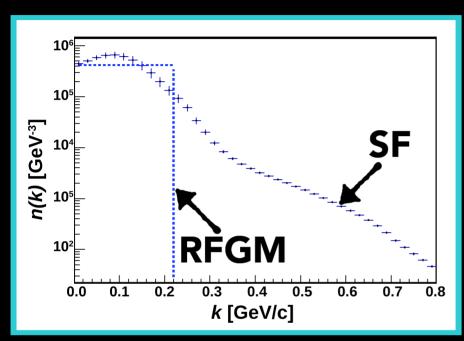


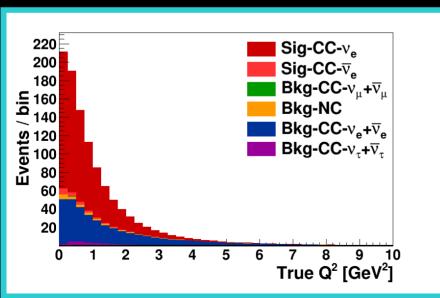
Example: NuMI 40

Cross Section Models

- Largest uncertainties due to absorption of nuclear model uncertainties
- Need to understanding the roll of nuclear dynamics on the low-Q2 region
 - Diagrams involving MEC, 2p2h, etc
 - Distinguishing effects from FSI experimentally
 - Models starting to work their way into generators
- Single pion production uncertainties seem to be converging
 - Explanations for differences between data sets have been provided
 - Still questions about transition to DIS region, and contributions from and interference with "non-resonant backgrounds"
- DIS interaction uncertainties are dominated by low-W hadronization models
- Coherent interactions are not well constrained, but make only a small contribution
- The ν_e and ν_τ cross sections have not been (well) measured
 - $\sigma(\nu_e)/\sigma(\nu_\mu)$ is unknown at low energies; may be an issue between 0.5-1.5 GeV (~2nd Max)
 - $-\sigma(v_{\tau})/\sigma(v_{u})$ error related to cross section terms prop to lepton mass
- $\sigma(\overline{v})/\sigma(v)$ errors are related to FSI

Nuclear Models: Nuclear Initial State





- RFG assumes no nucleonnucleon interactions
- These interactions allow correlated states
- Changes nucleon momentum probability densities
 - Important at low Q²
 - Exp: Spectral Functions (SF)
- Also introduces new targets
 - Meson exchange currents
 - 2-partlice / 2-hole states
 - Cross section ~20-30% of QE
- Contribution and uncertainties now covered by altering M_A^{QE}

Nuclear Models: Final State Interactions (FSI)

- Interactions of final-state particles with the nuclear medium
- Does not effect cross sections
- Changes hadronic shower and reconstructed quantities
 - Calorimetric energy estimators
 - Signal/Background acceptances
- Energy spectra are convolutions
 - Flux, Cross section, Detector effects, FSI, and Oscillations
 - Difficult to disentangle
 - Different for \overline{v} and v; different y_{bi}

- The good news
 - Not neutrino/weak physics
 - Can study with external data
 - Large detailed data sets
 - Several working models of various complexity
- The bad news
 - It's QCD
 - It alters observables
 - Convoluted with other sources of E_v uncertainty
 - Relative \overline{v}/v uncertainties currently provide freedom to mimic δ_{cp} -like effects

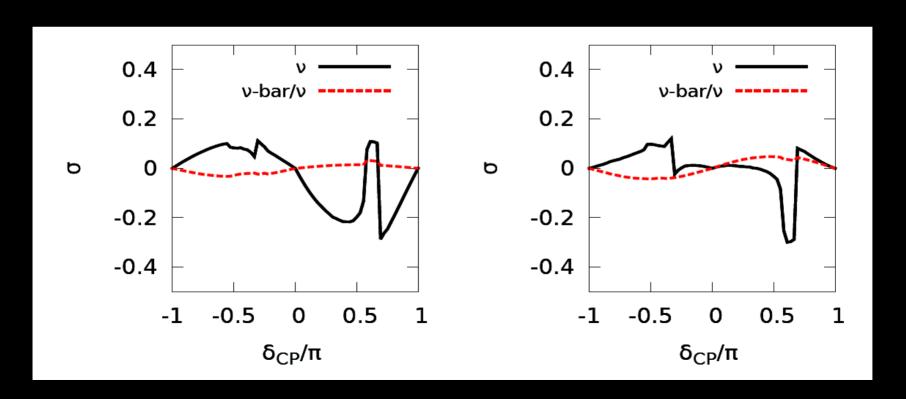
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Detector Response and Reconstruction

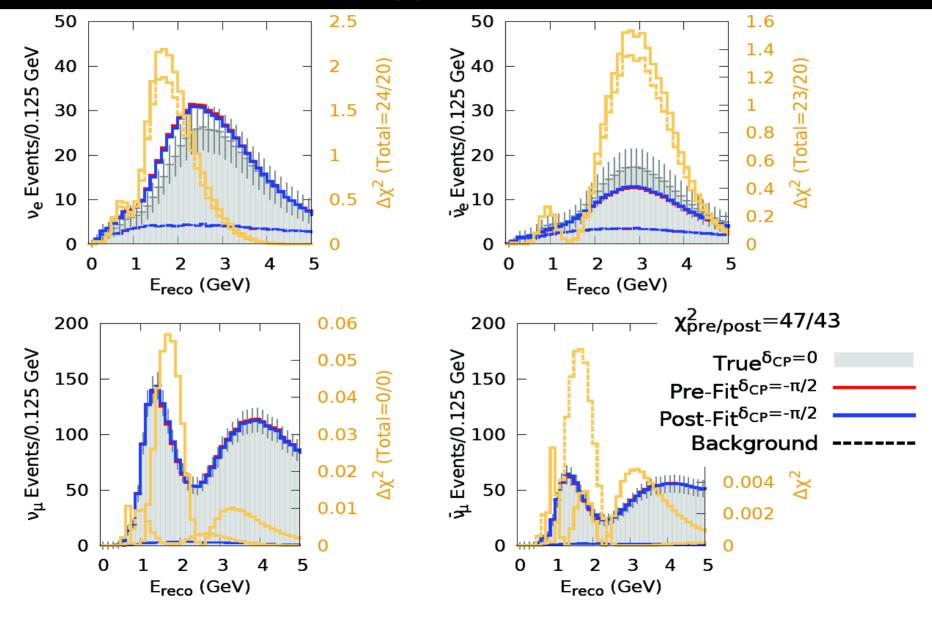
- The energy scale estimates the ν energy from the charge deposition in the detector; for DUNE:
 - 1) Reconstruct energy for tracked particles
 - 2) Estimate energy deposition from "hadronic shower fuzz"
 - 3) Correct for missing energy from neutral particles (mostly neutrons)
 - Mistakes in any step (esp step 3) can induce a bias
 - It is also important to accurately estimate the spread about the mean which determines the energy resolution
 - Particles of the same type and energy do not deposit identical amounts of charge
 - Secondary interactions can alter charge deposition rates and patterns
 - FSI can alter the flavor and momentum of particles exiting the interaction vertex
- Energy scale systematics can be dangerous because they can shift the reconstructed energy peak, and induce different responses for v and \overline{v}

Pull Terms for CC M_A res & CC M_A QE

- When both M_A^{res} & M_A^{QE} are allowed to vary the behavior of the pulls becomes more complex
- Large fluctuation around "inflection points" is enhanced
- Still constrained within $\sim 0.2\sigma$



CPV Fit Spectra and χ^2 with Variations in M_A^{res} (w/ osc systs)



CPV Fit Spectra and χ^2 with Variations in M_{Δ}^{res} & M_{Δ}^{QE}

